Impact of small scale rainfall uncertainty on urban discharge forecasts

A. GIRES[1], D. SCHERTZER[1], I. TCHIGUIRINSKAIA[1], S. LOVEJOY[2], C. ONOF[3], C. MAKSIMOVIC[3] & N. SIMOES[3,4]

1 *Université Paris-Est Ecole des Ponts ParisTech LEESU, 6-8 Av Blaise Pascal Cité Descartes, Marne-la-Vallée, 77455 Cx2, France*
auguste.gires@leesu.enpc.fr

2 *McGill University, Physics Department, Montreal, PQ, Canada*

3 *Imperial College London, Department of Civil and Environmental Engineering, UK*

4 *Department of Civil Engineering, University of Coimbra, Coimbra, Portugal*

Abstract We used a multifractal characterization of two heavy rainfall events in the London area to quantify the uncertainty associated with the rainfall variability at scales smaller than the usual C-band radar resolution (1 km^2 × 5 min) and how it transfers to sewer discharge forecasts. The radar data are downscaled to a higher resolution with the help of a multifractal cascade whose exponent values correspond to the estimates obtained from the radar data. A hundred downscaled realizations are thus obtained and input into a semi-distributed urban hydrological model. Both probability distributions of the extremes are shown to follow a power-law, which correspond to a rather high dispersion of the results, and therefore to a large uncertainty. We also discuss the relationship between the respective exponents. In conclusion, we emphasize the corresponding gain obtained by higher resolution radar data.

Key words multifractals; rainfall downscaling; urban hydrology; power law

INTRODUCTION

This paper implements multifractal techniques (see Lovejoy & Schertzer, 2007 for a recent review), which are standard tools to analyse and simulate geophysical fields, e.g. rainfall, that are extremely variable over a wide range of scales, and in urban hydrology to quantify the impact of small scale unmeasured rainfall variability. Indeed numerous hydrological studies (see Singh, 1997 for a review) show that rainfall variability has an impact on the modelled flows, which is more or less significant according to the rainfall event and the catchment size and features. In urban areas the effects are enhanced because of greater impervious coefficients and shorter response times (Aronica & Cannarozzo, 2000; Segond *et al*., 2007).

In this paper we study the mainly urban 910-ha Cranbrook catchment, situated in the London Borough of Redbridge, and known for regular local flooding (Gires *et al.,* 2010b). The rainfall data are obtained from the Nimrod composites, a radar product of the Met Office in the UK (Harrison *et al*., 2000), whose resolution is 1 km in space and 5 min in time. Here a winter frontal rainfall event (9 February 2009) and a summer convective one (7 July 2009) are investigated. Square areas of size 64 km^2 during 21 hours, centred on the heaviest rainfalls of these events, are analysed. Figure 1 displays the total rainfall depth for both events. The very localized rainfall cells of the convective July event are clearly visible.

In the next sections, first the multifractal properties of the rainfall fields are analysed for both events. Then an ensemble of realistic spatially downscaled (to the scale of 125 m) rainfall fields is generated with the help of universal multifractal cascades, and the corresponding ensemble of hydrographs is simulated. The variability among these ensembles is used to characterize the uncertainty due to small-scale unmeasured rainfall variability, mainly on the peak flow.

MULTIFRACTALS AND RAINFALL DOWNSCALING

In this section, the multifractal analysis of the rainfall events and the implemented downscaling technique are briefly presented. More details can be found in Gires *et al*. (2010b).

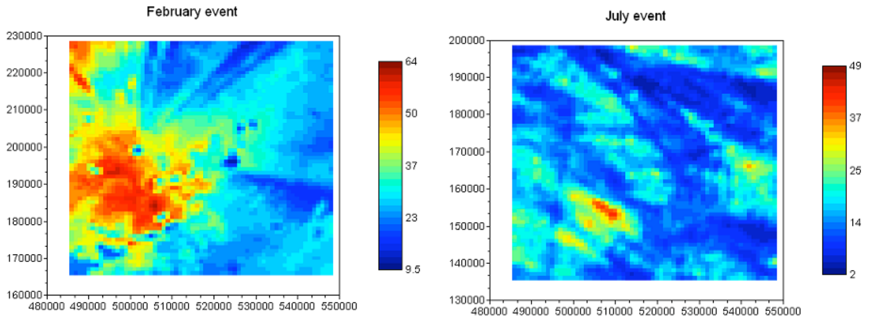

Fig. 1 Map of the total rainfall depth (in mm) over the studied area for the February (left) and July (right) events. The coordinate system is the British National Grid (unit: m)

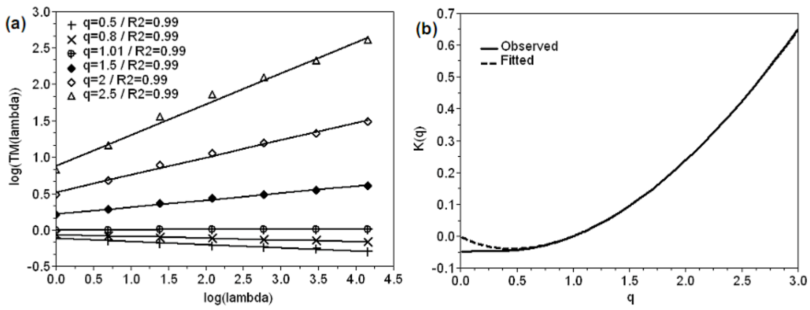

Fig. 2 For the February event: (a) definition of the scaling moment function (equation (1)) and (b) plot of $K(q)$.

Multifractals

In this paper the rainfall field is investigated with the help of universal multifractals, which have been extensively used to analyse geophysical fields that are extremely variable over a wide range of scales (Lovejoy & Schertzer, 2007; Royer *et al.*, 2008; Nykanen, 2008; Gires *et al.*, 2010a). They basically rely on the concept of multiplicative cascades. In that framework the statistical moment of arbitrary qth power of the rainfall field R_λ at the resolution λ (=L/l, the ratio between the outer scale of the phenomenon and the observation scale) exhibits a scaling behaviour:

$$< R_\lambda{}^q > \approx \lambda^{K(q)} \tag{1}$$

where < > denotes ensemble average (over the different time steps analysed), and ≈ asymptotic equivalence. $K(q)$ is the scaling moment function and quantifies the scaling variability of the rainfall field. Figure 2(a) displays equation (1) in a log-log plot for the February event. The straight lines (the coefficient of determination are all >0.99), whose slopes are $K(q)$, indicate a good scaling behaviour and show the relevance of this analysis. Similar curves are found for the July event. $K(q)$ is plotted Fig. 2(b).

In the specific framework of universal multifractals (Schertzer *et al.*, 1997), $K(q)$ is described by three scale independent parameters (UM parameters): C_1 the mean intermittency ($C_1 = 0$ for a uniform field), α the multifractality index ($\alpha = 0$ for a monofractal field, and $\alpha = 2$ for the extreme log-normal case), and H the non-conservation ($H = 0$ for a conservative field). $K(q)$ is given by:

$$K(q) = \frac{C_1}{\alpha - 1}\left(q^\alpha - q\right) + Hq \tag{2}$$

Greater values of C_1 and α correspond to strong extremes. The parameters are estimated with the help of the DTM technique and spectral slope (Lavallée *et al.*, 1993). Here the numerical values of the UM parameters are quite different for both event. The estimates of α, C_1 and H are 1.62, 0.14

and 0.56, respectively, for the February event and 0.92, 0.49 and 0.57, respectively, for the July event. $K(q)$ plotted with the estimated UM parameters for the February event is displayed in Fig. 2(b). The agreement with the empirical curve is very good, and the discrepancies for small moments are explained by a multifractal phase transition associated to the influence of the numerous zeros (i.e. a pixel of a time step with no rain) (Gires *et al.*, 2010a). The rainfall events exhibit two different statistical behaviours (Hubert *et al.*, 1993): indeed for the February event $\alpha > 1$ which indicates that the extreme values are not bounded whereas for the July event $\alpha < 1$ indicates bounded extreme values. In the multifractal framework, the probability distribution of the extreme values observed on a given dataset are expected to follow a power-law (or equivalently, the statistical moments cannot be estimated for moments greater than the power law exponent) whose exponent strongly depends on C_1. Here C_1 for the July event is more than three times greater than C_1 for the February event, indicating a lower theoretical exponent. The effect is partially compensated by the greater value of α for the February event. This means that extreme values will be observed more frequently for the July event.

Multifractal spatial downscaling of the rainfall field

A stochastic downscaling technique is used to generate realistic high-resolution rainfall fields (Venugopal *et al.*, 1999; Deidda, 2000; Olsson *et al.*, 2001; Ferraris *et al.*, 2002; Rebora *et al.*, 2006). The framework of the cascade process is well suited to this problem (Biaou *et al.*, 2003), since continuing the cascade beyond the observation resolution enables generating a realistic high-resolution rainfall field. More precisely, for each pixel of 1 km × 1 km three steps of discrete UM cascades (Pecknold *et al.*, 1993) are simulated with the UM parameters estimated on the available data (i.e. on a range of scale from 1 to 64 km). The final resolution is 125 m × 125 m. This process is illustrated in Fig. 3(a). An example of downscaling (which generates rainfall variability inside a radar pixel) over the modelled area for a time step of the February event is displayed in Fig. 3(b). The downscaling of two consecutive time steps is independent, but the obtained variables remain dependant because they are generated from larger structures that are dependent.

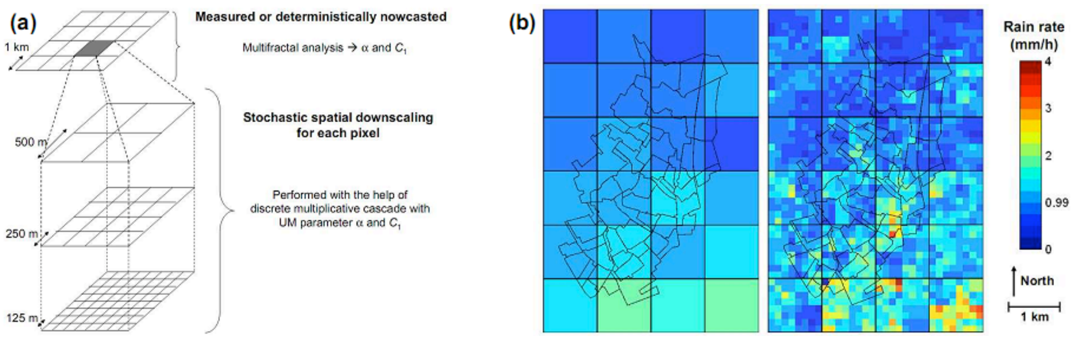

Fig. 3 Illustration (a) and example over the Cranbrook area for a time step of the February event (b) of the downscaling technique implemented.

RAINFALL–RUNOFF MODEL

For the Cranbrook catchment, Infoworks CS (Wallingford Software, 2009) was used by Thames Water Utilities (2002) to calibrate a semi-distributed (i.e. the area is divided into 51 sub-catchments of size ranging from 1 to 62 ha) model that includes the major surface water sewers (Fig. 4). Sub-catchments are defined by sewer nodes and are considered as being homogeneous. Their discharge is computed from the rainfall with the help of a double linear reservoir model. The main parameters are the slope and the length (represented with dashed lines in Fig. 4). The simulation parameters were maintained unchanged for all simulations. The total rainfall depth over the 51 sub-catchments for the

July event is also displayed in Fig. 4. It ranges from 4 to 14 mm, with the heaviest rainfall situated in the south near the outlet. A similar distribution with values ranging from 16 to 23 mm is observed for the February event. Conduits drain the water from one (for upstream conduits) or several sub-catchment(s) (for downstream conduits), and their characteristic length L_{da} is defined as the square root of the area drained by the conduit. In this paper the hydrographs of 10 conduits with L_{da} ranging from 370 to 2910 m are analysed (Fig. 4). This enables us to study the impact of the size of the studied area on the uncertainty associated with small scale rainfall variability.

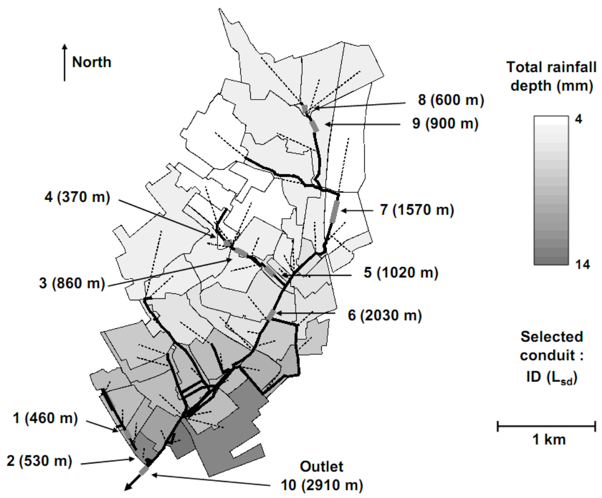

Fig. 4 The Cranbrook Catchment and its underground sewer system. The total rainfall depth over the 51 sub-catchments for the July event is also displayed.

QUANTIFYING THE IMPACT OF SMALL-SCALE UNMEASURED RAINFALL VARIABILITY

An ensemble of 100 realistic rainfall downscaled fields and the corresponding ensemble of hydrographs were simulated. The uncertainty associated with small-scale unmeasured rainfall variability is quantified by investigating the variability among the ensembles.

Variability among the ensemble of rainfall fields

To give an insight of the uncertainty on the rainfall fields, for each conduit we evaluated the maximum average rainfall intensity (R_{max}) over the drained area for each sample of downscaled fields. The histogram of these values (one per sample) for the conduit 2 and the February event is shown in Fig. 5(a). Similar curves are found for the other conduits and for the July event. It appears that the extreme cases (i.e. the right part of the histogram) exhibit a power-law behaviour of the form:

$$\Pr(R_{\max} > x) \approx x^{-k} \tag{3}$$

Indeed, this relation in a log-log plot is displayed in Fig 5(b), and the determination coefficient of the straight line describing the fall-off of the probability distribution is very good (0.98 ± 0.01 and 0.97 ± 0.01 for the February and the July event, respectively, according to the conduit). The values k_{rain} found are plotted against L_{da} in Fig. 6. First it appears that k_{rain} increases with L_{da}, which implies a thinner probability fall-off for larger drained area. It means that the effect of small-scale rainfall variability is damped for larger area, which was expected. Second, in general k_{rain} is smaller for the July event, which reflects the fact that the variability among the ensemble of downscaled rainfall fields is greater for the convective event than for the frontal event, which was expected due to greater values of C_1.

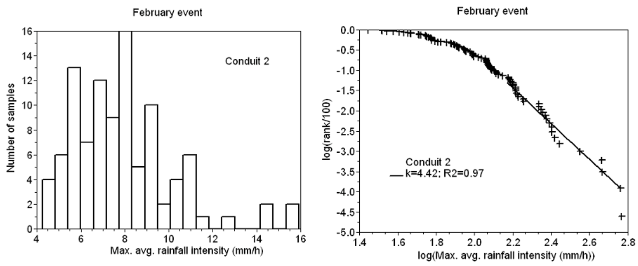

Fig. 5 Histograms (right) and determination curve of k_{rain} for the maximum average rainfall rate of the conduit 2 for the February event.

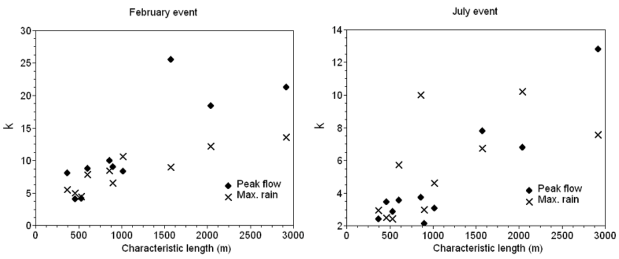

Fig. 6 k_{rain} and k_{flow} vs. L_{da} for the February event (right) and the July event (left).

Variability among the ensemble of simulated peak flow

A similar analysis was performed on the ensemble of simulated hydrographs. For each selected conduit, the peak flow and its time of occurrence was retrieved. The first point is that no significant influence was found on the time of occurrence. Concerning the peak flow, as for the maximum average rainfall, the probability distribution exhibits power-law fall-off (curves similar to Fig. 5(b) but for the peak flow are found, with R^2 equal to 0.97 ± 0.01 for both events according to the conduit). The power-law behaviour means that the uncertainty on the simulated peak flow associated to unmeasured small-scale rainfall variability cannot be neglected. It is striking to see that the uncertainties are significant despite the coarse resolution of the hydrological model (the average square root of sub-catchment area is 380 m) with regard to the resolution of the downscaled rainfall fields (125 m). Nevertheless further investigations with hydrological models with greater spatial resolution and taking into account the interaction between surface flow and the sewer system (El Tabach *et al.*, 2009; Maksimovic *et al.*, 2009) would be needed to fully take advantage of the spatial downscaling. Figure 6 displays the exponent k_{flow} *vs* the characteristic length of the conduit L_{da} for both events. As for k_{rain}, k_{flow} tends to increase with L_{da}. Independently of the event and L_{da}, the variability among the ensemble of rainfall fields is transferred to the ensemble of peak flow with the same qualitative features (i.e. a power-law fall-off of the probability distribution). Quantitatively it appears that k_{flow} is often (i.e. most of the conduits for the February event and some for the July event) greater than k_{rain}, which would mean that the rainfall–runoff process slightly dampens the variability of the ensemble of rainfall fields. Finally it appears that the variability observed for the July event is similar to the one observed up to 1 km for the February event.

CONCLUSION

Multifractal cascades are used to generate an ensemble of realistic downscaled rainfall fields of a convective and a frontal rainfall event (after validating this framework for both events). The probability distribution of the generated rainfall extremes exhibits a power-law fall-off, which is also retrieved on the peak flow of the corresponding simulated ensemble of hydrographs. It means that the uncertainty associated with small-scale rainfall variability cannot be neglected. As a consequence it is recommended to either take into account this uncertainty in the real-time management of sewer networks or to improve the resolution of rainfall data in urban areas by implementing X-band radars whose spatial resolution is roughly 100 m. Concerning the numerical values of the characteristic exponents of the power-law fall-off it seems that they are greater for peak flow than for rainfall, and for larger areas. They are greater for the frontal event than for the convective one. Nevertheless further investigations with other case studies and rainfall events are required to clarify this point.

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